

A new invariant for virtual knots and forbidden moves

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1. Introduction

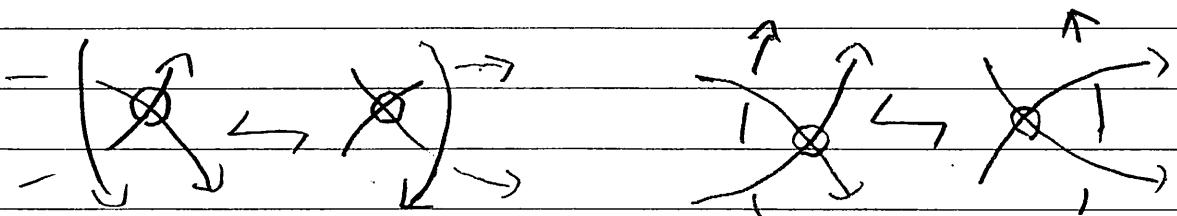
スライド参照 (別紙)

Theorem 1. ([Kanenobu], [Nelson])

forbidden move は unknotting operation
である。

* k: virtual knot $\leftarrow \leftarrow \dots \leftarrow 0$

Lemma 2. ori. forbidden move は
下図で実現可能



sketch of proof

$$\textcircled{1} - \neg(X \leftarrow X) \rightarrow$$

11

$$\textcircled{2} - \neg(X \leftarrow X) \rightarrow$$

12

$$\textcircled{3} \neg(X \leftarrow X) \rightarrow$$

$$\textcircled{4} \neg(X \leftarrow X) \rightarrow$$

$$\textcircled{5} \neg(X \leftarrow X) \rightarrow$$

13

Lemma FTM(2)

$$\boxed{\textcircled{6}} - \neg(X \leftarrow X) \rightarrow$$

14

$$\textcircled{7} \neg(X \leftarrow X) \rightarrow$$

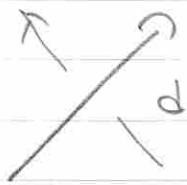
$$\textcircled{8} \neg(X \leftarrow X) \rightarrow$$



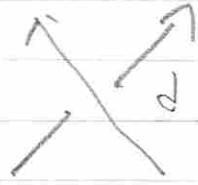
- k : an oriented virtual knot diagram.

$$c(k) := \{ \text{All real crossings of } k \}$$

$d \in c(k)$



$$\text{sign}(d) = +1$$



$$\text{sign}(d) = -1$$

$\tilde{k}^d = \tilde{k}_1^d \cap \tilde{k}_2^d : k \in d$ is smoothing left diagram の flat virtual diagram

$$\tilde{k}_1^d \cap \tilde{k}_2^d = \{ \text{All flat crossing between } \tilde{k}_1^d \text{ & } \tilde{k}_2^d \}$$

$e \in \tilde{k}_1^d \cap \tilde{k}_2^d$

$\text{sgn}(e) = 1$ $\text{sgn}(e') = -1$

intersection index $i(d) = \sum_{e \in \tilde{k}_1^d \cap \tilde{k}_2^d} \text{sgn}(e)$
 ([A. Henrich])

lemma 3. ([A. Henrich]).

$\hat{L} = \hat{k}_1 \cup \hat{k}_2$: an ori. 2-comp. flat

$\hat{k}_1 \cap \hat{k}_2 := \{ \text{All flat crossing between } \hat{k}_1 \text{ & } \hat{k}_2 \}$

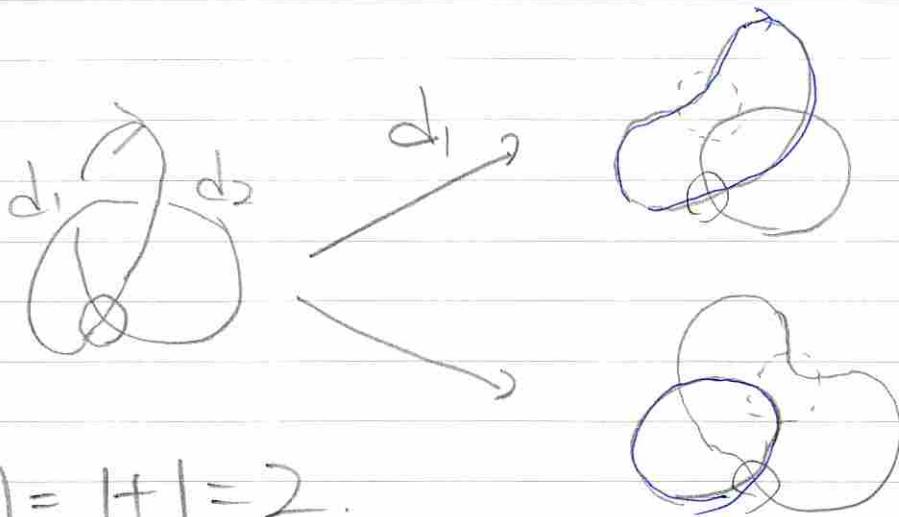
$$i(L) := \sum_{e \in \hat{k}_1 \cap \hat{k}_2} \operatorname{sgn}(e).$$

$|i(L)|$ is 2-comp. flat virtual link inv.

Definition

$$P(K) = \sum_{d \in c(K)} \operatorname{sign}(d) |i(d)|.$$

例



$$P(K) = 1 + 1 = 2.$$

Theorem 5

 $P(K)$ は virtual knot の inv.

(?) Lemma 3 ①), QRM で 不変を示す。

Theorem 6

 K, K' : virtual knots.s.t. $K \leftarrow K'$

$$P(K) - P(K') = 0. \quad \text{I2.}$$

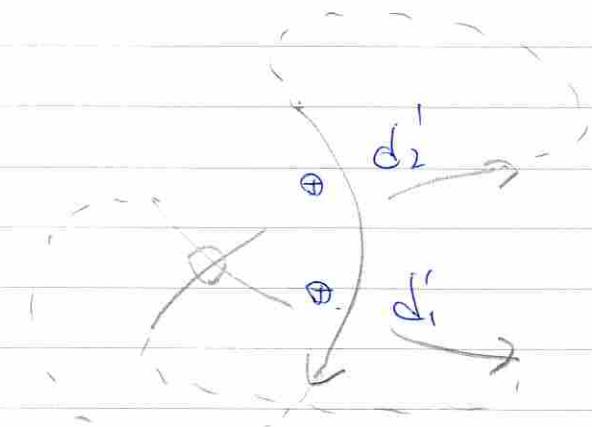
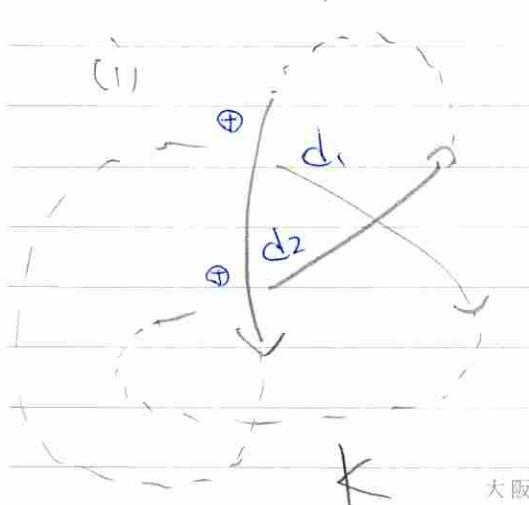
Corollary 7.

 K, K' : virtual knots.

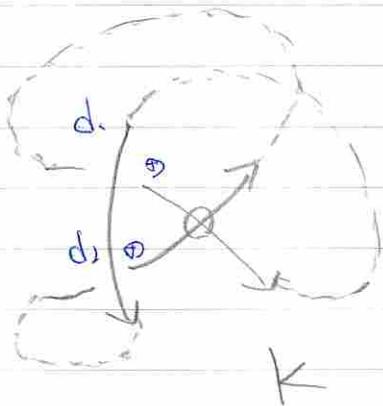
$$d_F(K, K') \geq \frac{|P(K) - P(K')|}{2}$$

sketch proof. Lemma 2 ①).

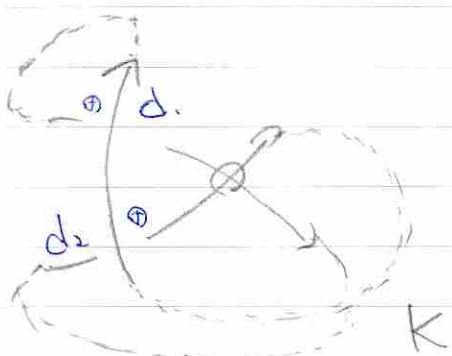
(1)



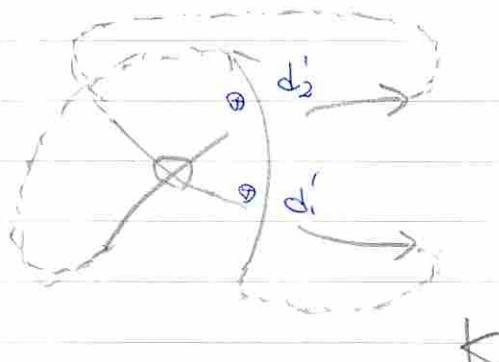
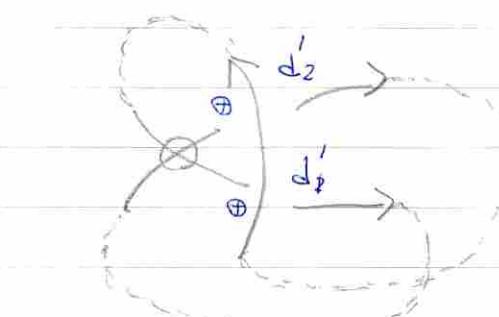
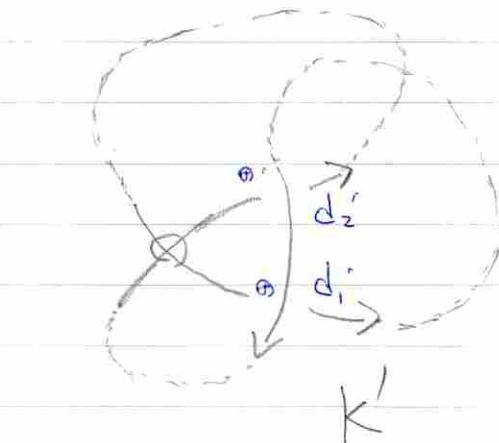
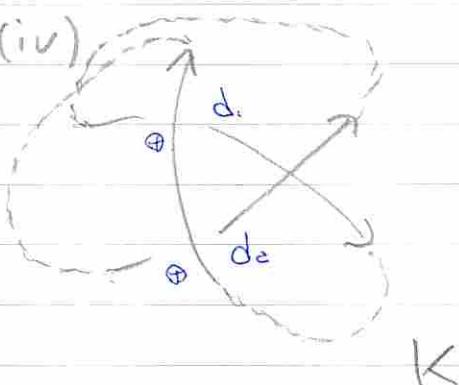
(ii)



(iii)



(iv)



$c \in C(K) \setminus \{d_1, d_2\}$

$c' : c \mapsto K'$'s crossing.

step1 $c + c'$ 得す43項

- $\text{sgn}(c) = \text{sign}(c')$

- $|i(c)| = |i(c')|$

$$\left(\begin{array}{l} \because \tilde{d}_i \in \tilde{k}_1^c \cap \tilde{k}_2^c \\ \Rightarrow \text{sgn}(\tilde{d}_i) = \text{sgn}(\tilde{d}'_i) \quad (i=1,2) \end{array} \right)$$

$\rightarrow \text{sign}(c) |i(c)| = \text{sign}(c') |i(c')|$

step2 $d_1, d_2 + c$ 得す43項

- $\text{sign}(d_i) = \text{sign}(d'_i) = +1$

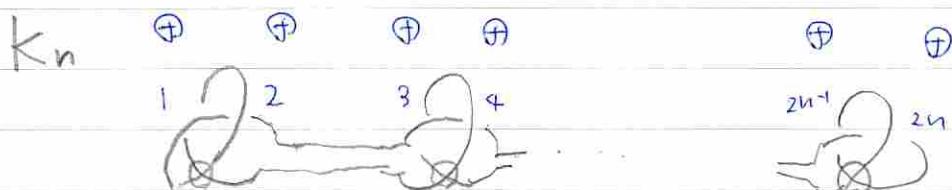
- $|i(d_i)| = |i(d'_i)| \pm 1$

$$\left(\begin{array}{l} \text{④ } \tilde{d}_j \in \tilde{k}_1^{d_i} \cap \tilde{k}_2^{d_i} \Leftrightarrow \tilde{d}'_j \notin \tilde{k}_1^{d'_i} \cap \tilde{k}_2^{d'_i} \\ j \neq i, j=1,2 \end{array} \right)$$

$$|\text{sign}(d_1) |i(d_1)| + |\text{sign}(d_2) |i(d_2)|$$

$$- |\text{sign}(d'_1) |i(d'_1)| + |\text{sign}(d'_2) |i(d'_2)|$$

Example 8.

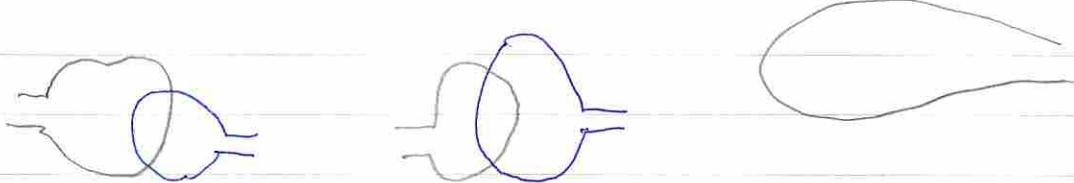


1)



1)

$$\ell = 1, \dots, n$$



$$|i(2\ell-1)| = 1$$

$$\text{sign}(2\ell-1) = 1$$

$$|i(2\ell)| = 1$$

$$\text{sign}(2\ell) = 1$$

$$P(K_n) = \sum_{k=1}^{2n} 1 = 2n$$

$$U_F(K_n) = n$$