Rmk. L : a link in S^3

• $\overline{\mu}_L(i_1 i_2 \cdots i_m) = \overline{\mu}_L(i_2 \cdots i_m i_1)$ (Cyclic symmetry)

• $lk(L) = 0 \Rightarrow \overline{\mu}_L(\underline{i_1 i_2 i_3}) = -\overline{\mu}_L(\underline{i_2 i_1 i_3})$

cyclic symmetry より   permutation

• $\overline{\mu}_L(I) \equiv \mu(I) \mod \Delta_L(I) = \gcd\{\mu(J) \mid J : \text{a subseq. of } I\}$

で計算可能である.

☺ $|I| = m \ (\geq 3)$, induction on m

$m = 3 \Rightarrow lk(ij) = lk(ji)$ より O.K.

$m > 3$ のとき $\Delta_L(i_1 \cdots i_m) = \gcd\{\mu(J) \mid J : \text{a subseq. of } i_1 \cdots i_m\}$

が成り立つと仮定する.

$\Delta_L(i_1 \cdots i_{m+1})$

$= \gcd\left\{\begin{array}{l} \mu(i_1 \cdots i_m i_{m+1}), \ \mu(i_2 \cdots i_m i_1), \cdots, \mu(i_m i_1 \cdots i_{m-1}), \ \Delta_L(i_1 \cdots i_m \check{i}_{m+1}) \\ \mu(i_1 \cdots \check{i}_m i_{m+1}), \ \mu(i_2 \cdots i_{m-1} i_{m+1} i_1), \cdots, \mu(i_{m+1} i_2 \cdots i_{m-1}), \ \Delta_L(i_1 \cdots \check{i}_m i_{m+1}) \\ \quad \vdots \\ \mu(\check{i}_1 i_2 \cdots i_{m+1}), \ \mu(i_3 \cdots i_{m+1} i_2), \cdots, \mu(i_{m+1} i_2 \cdots i_m), \ \Delta_L(\check{i}_1 i_2 \cdots i_{m+1}) \end{array}\right\}$

$= \gcd\left\{\begin{array}{l} \overline{\mu}(i_1 \cdots i_m \check{i}_{m+1}), \ \Delta_L(i_1 \cdots i_m \check{i}_{m+1}), \\ \quad \vdots \\ \overline{\mu}(\check{i}_1 i_2 \cdots i_{m+1}), \ \Delta_L(\check{i}_1 i_2 \cdots i_{m+1}) \end{array}\right\} = \gcd\left\{\begin{array}{l} \mu(i_1 \cdots i_m \check{i}_{m+1}), \ \Delta_L(i_1 \cdots i_m \check{i}_{m+1}) \\ \quad \vdots \\ \mu(\check{i}_1 i_2 \cdots i_{m+1}), \ \Delta_L(\check{i}_1 i_2 \cdots i_{m+1}) \end{array}\right\}$

• $\overline{\mu}_L(I) = \overline{\mu} \bigcup_{i_\varrho \in \{I\}} K_{i_\varrho}(I)$ $\left(I = i_1 \cdots i_m, \ \{I\} = \{i_1, \cdots, i_m\}\right)$

• L' : a link in S^3 obtained from L by taking zero framed parallels of the comp. of L

the ith comp. of $L' \leftrightarrow$ the $h(i)$th comp. of L

$\overline{\mu}_{L'}(i_1 i_2 \cdots i_m) = \overline{\mu}_L(h(i_1) h(i_2) \cdots h(i_m))$

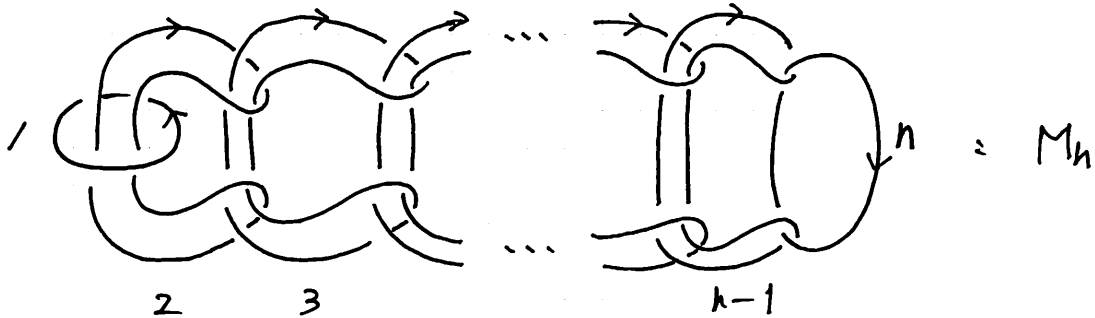$\overline{\mu}_{L'}(12345)$ $=$ $\overline{\mu}_L(11233)$

L' L

Thm (Milnor '54)

M_n : the n-comp. Milnor link (or the Sutton Hoo link)

Then the $\overline{\mu}$-inv. of the length $\leq n-1$ vanish, and

$$\overline{\mu}_{M_n}(1\, i_2\, i_3 \cdots i_{n-1}\, n) = \begin{cases} 1 & (i_2\, i_3 \cdots i_{n-1} = 23 \cdots n-1) \\ 0 & (\text{otherwise}) \end{cases}$$
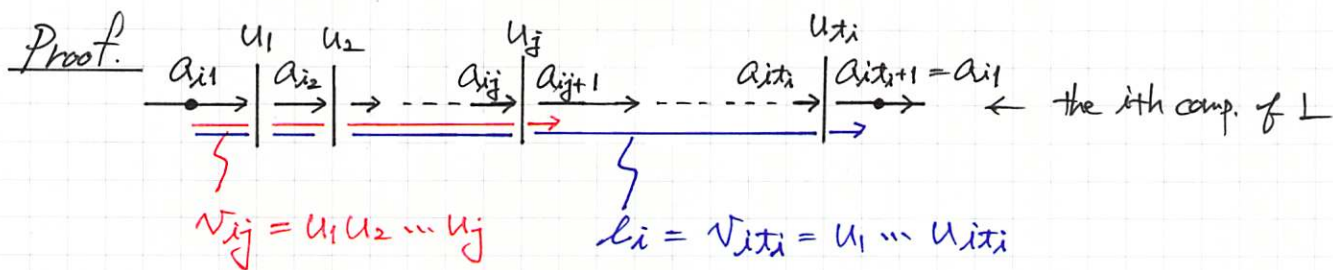

$= M_n$

Rmk $\quad \overline{\mu}_{M_n}(1\, i_2 \cdots i_{n-1}\, n) = \mu_{M_n}(1\, i_2 \cdots i_{n-1}\, n)$

§ Proof of Chen's theorem

Thm $G/G_q \cong \langle \alpha_1, \cdots, \alpha_n \mid [\alpha_i, \lambda_i] \ (i=1,\cdots,n), \ A_q \rangle$

Proof.

$$\overset{u_1}{\underset{a_{i1}}{\longrightarrow}} \Big| \overset{u_2}{\underset{a_{i2}}{\longrightarrow}} \Big| \longrightarrow \cdots \longrightarrow \overset{u_j}{\underset{a_{ij}}{\longrightarrow}} \Big| \underset{a_{ij+1}}{\longrightarrow} \longrightarrow \cdots \longrightarrow \overset{u_{t_i}}{\underset{a_{it_i}}{\longrightarrow}} \Big| \underset{a_{it_i+1}=a_{i1}}{\longrightarrow} \quad \leftarrow \text{ the ith comp. of } L$$

$$N_{ij} = u_1 u_2 \cdots u_j \qquad \ell_i = N_{it_i} = u_1 \cdots u_{it_i}$$

$G = \pi_1(S^3 \setminus L) \cong \langle a_{ij} \mid r_{ij} = a_{ij+1}^{-1} u_j^{-1} a_{ij} u_j \rangle$: the Wirtinger pres.

こ の と き $S_{ij} = a_{ij+1}^{-1} N_{ij}^{-1} a_{11} N_{ij}$ と お く と

$$G = \langle a_{ij} \mid r_{ij} \rangle \cong \langle a_{ij} \mid S_{ij} \rangle = \overline{A}/N = \mathrm{Con}(\{S_{ij}\} ; \overline{A})$$

$$\text{(i)} \begin{cases} r_{i1} = S_{i1} \\ r_{ij} = S_{ij}(u_{ij}^{-1} S_{ij-1}^{-1} u_{ij}) \quad \text{for } 1 < j \le t_i \end{cases} \quad \text{な の で}$$

Tietze 変換 よ り

$$\langle a_{ij} \mid S_{ij} \rangle \overset{(T1)}{\cong} \langle a_{ij} \mid S_{ij}, r_{ij} \rangle$$

逆 に $S_{ij} = r_{ij} u_{ij}^{-1} r_{ij-1} u_{ij-1}^{-1} \cdots r_{i2} u_{i2}^{-1} \cdot r_{i1} \cdot u_{i2} u_{i3} \cdots u_{ij}$ な の で

Tietze 変換 よ り

$$\langle a_{ij} \mid r_{ij} \rangle \overset{(T1)}{\cong} \langle a_{ij} \mid r_{ij}, S_{ij} \rangle \quad /\!/$$

cf. Tietze 変換　　　　　　　　　　　　　　　 帰結群

(T1) $\langle X \mid R \rangle \cong \langle X \mid R, \{r\} \rangle \quad (r \in \mathrm{Con}(R ; X))$

(T2) $\langle X \mid R \rangle \cong \langle X, \{y\} \mid R, \{y w^{-1}\} \rangle \quad \begin{pmatrix} y \notin X \\ w \in \langle X \rangle \end{pmatrix}$